# Pathways to Increasing the Power of Possibilities in Math

Jacob S. Gray[1], Ph.D.
Courtney E. Wheeler[2], Ph.D.
[1]Acadience Learning
[2]University of Minnesota

## IMPROVED GOAL-SETTING WITH PATHWAYS OF PROGRESS™

When used in conjunction with the Acadience® Math benchmark goals, Pathways of Progress further empowers educators to set goals that are meaningful, ambitious, and attainable. The Acadience Math benchmark goals are the same for all students in a grade, regardless of their starting skill level, and represent the lowest score for which a student is likely to still be on track to reach future math outcomes (e.g., to be on track for third grade, every second-grade student should reach a Acadience Math Composite Score (MCS) of 66 by the end of the year).

While benchmark goals are meaningful, there may be some students for whom they are not ambitious enough, and others for whom they are unattainable. Pathways of Progress helps increase decision-making precision with respect to goal setting and evaluating progress. Pathways of Progress allows teachers to use a normative context, in addition to the benchmark goals, when setting goals and evaluating progress. Pathways of Progress clarifies what rate of progress is Typical, Above Typical, or Well Above Typical. Pathways of Progress also informs educators when the rate of progress is Below Typical or Well Below Typical.

## PATHWAYS OF PROGRESS LOGIC AND METHODOLOGY – HOW PATHWAYS WERE CREATED

Pathways of Progress emphasizes the end point of the pathway and provides a normative framework for comparison when evaluating student progress. Student progress is evaluated relative to the student's peers, that is, growth is compared to students with similar initial skills at the same grade level on the same material. Progress that is Typical or Above Typical is considered attainable progress. Progress that is Above Typical or Well Above Typical can be considered ambitious progress.

The Pathways of Progress borders were calculated using spline regression with the beginning-of-year MCS in a multistep process described in the table below. Spline regression accounted for the majority of statistical considerations within the data, such as nonlinear relationships between the beginning-of-year and end-of-year individual Acadience Math measures, heteroscedasticity, and sparse data at the lower-end and the upper-end of the beginning-of-year MCS distribution. The pathway borders were then checked for two conditions to ensure that the predicted scores were monotonic nondecreasing for the range of the beginning-of-year composite score. These two conditions were:

1. The predicted spline values must be monotonic nondecreasing with beginning-of-year MCS. Predicted scores that decrease with the beginning-of-year MCS that occurred at the high- or the low-end of the scoring distribution were replaced with the maximum or minimum predicted score, respectively, to keep the function monotonic nondecreasing. For predicted scores that decrease with beginning-of-year MCS in the middle of the scoring range, the value of lambda, i.e., the stiffness of the fit, was raised until the function was monotonic nondecreasing.
2. The predicted spline values must be positive. Negative predicted values were changed to zero.

### Statistical Analysis Steps

**Step 1.** For all grades, students were grouped together by their beginning-of-year MCS. For grades K–5, students were grouped for the scoring range between a composite score of 1 and the 99.5th percentile rank. For sixth grade, due to sparse data at the lower-end of the beginning-of-year MCS scoring range, students were grouped between the 1st and the 99th percentile rank. Next, for each unique beginning-of-year composite score, the 20th, 40th, 60th, and 80th quantiles were calculated for each end-of-year Acadience Math measure. This resulted in four new columns of data per measure.

Due to sparse data and changes in variance at the tails of the distribution of the beginning-of-year composite score, we did not evaluate outcomes based on scores at zero, above the 99.5th percentile rank of the grade-level beginning-of-year composite score for all grades, and below the .5th percentile rank for sixth grade.

**Table 1**
*Illustrative Example of End-of-year Third-Grade Acadience Math Composite Score Quantiles for a Selection of 10 Consecutive Beginning-of-Year Acadience Math Composite Scores.*

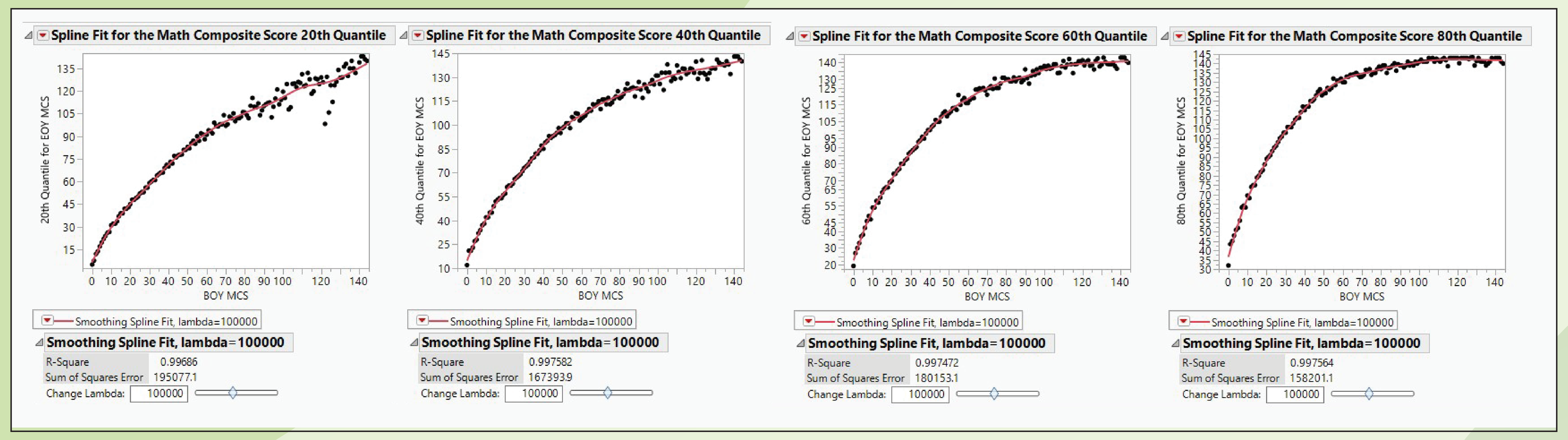
| BOY Math Composite Score | EOY Math Composite Score | | | | |
|---|---|---|---|---|---|
| | N | 20th Percentile | 40th Percentile | 60th Percentile | 80th Percentile |
| 16 | 2,844 | 35 | 50 | 65 | 86 |
| 17 | 2,834 | 36 | 52 | 67 | 88 |
| 18 | 3,047 | 38 | 54 | 69 | 90 |
| 19 | 2,868 | 40 | 55 | 71 | 92 |
| 20 | 2,950 | 41 | 57 | 73 | 94 |
| 21 | 3,062 | 43 | 59 | 74 | 96 |
| 22 | 2,995 | 44 | 60 | 76 | 97 |
| 23 | 2,968 | 45 | 61 | 78 | 99 |
| 24 | 2,913 | 47 | 63 | 79 | 101 |
| 25 | 3,075 | 48 | 64 | 81 | 102 |

*Note.* Scores for third-grade end-of-year Math Composite Score range from 0 to 208. Quantiles are calculated from the scoring range of beginning-of-year Match Composite Score between 1 and the 99.5th percentile rank (158).

## PATHWAYS OF PROGRESS LOGIC AND METHODOLOGY – HOW PATHWAYS WERE CREATED (Continued)

**Step 2.** A stiff spline regression model was fit to each quantile using beginning-of-year MCS as the predictor. There were four models per measure (i.e., one per quantile). Models were evaluated for goodness of fit via fit statistics and visual analytics.

**Figure 1**
*Spline Regressions for Math Pathways*



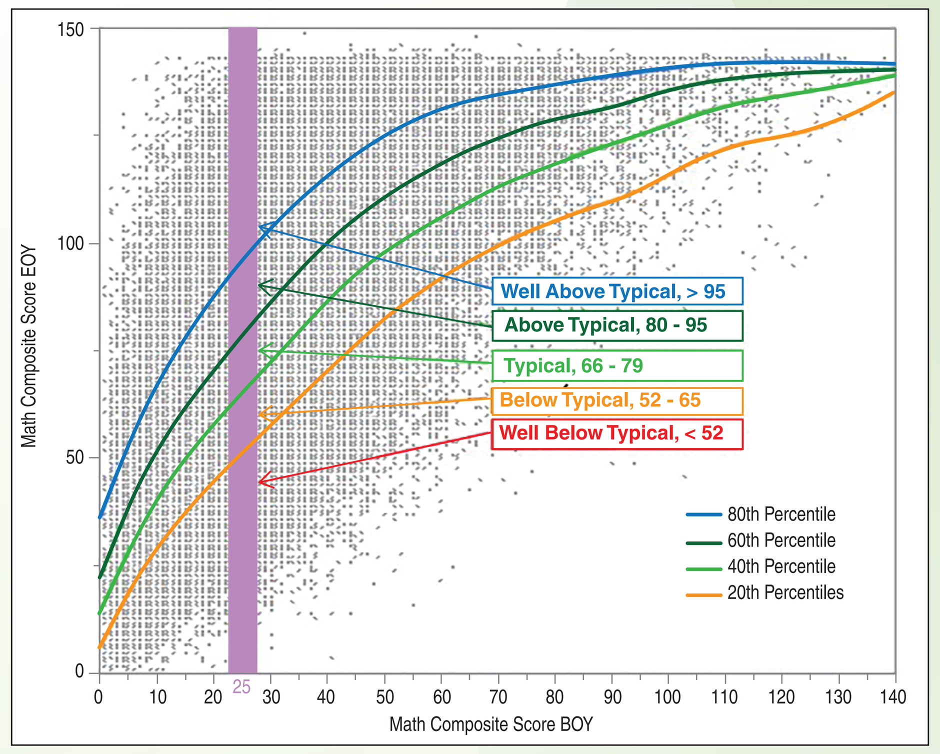
**Step 3.** The predicted quantile scores from the spline model were rounded to the nearest integer, and placed into a look-up table corresponding to each unique beginning-of-year MCS. For a score of zero, the predicted quantiles for when beginning-of-year MCS equals one was used. These resulting predicted quantiles are the pathway borders.

Within the borders, we define the rates of progress as shown in Table 2.

**Table 2**
*Illustration of Pathways of Progress Definitions*

| Quantile Range | Rate of Progress Description |
|---|---|
| Above 79% | Well Above Typical Progress |
| 60% - 79% | Above Typical Progress |
| 40% - 59% | Typical Progress |
| 20% - 39% | Below Typical Progress |
| Below 20% | Well Below Typical Progress |

**Figure 2**
*Pathways of Progress Example for Acadience Math*




## THE POWER OF PATHWAYS

### Methods

In order to assess the predictive validity of Pathways of Progress for Math, we examined the incremental validity that pathways offers when predicting later math outcomes. Logistic regression was used to predict the math skills as measured by Acadience Math at the end of the following grade. For example, the predictive power of pathways in kindergarten was assessed using a logistic regression that predicted students' MCS at the end of first grade. The first predictor was the beginning-of-year MCS in kindergarten, and then a second model was run that included the students' beginning-to-end of year pathways. The magnitude of predictive validity is examined as the additional variance explained by pathways.

**Table 3**
*Incremental Validity of Pathways of Progress for Math*

| Predictor | Outcome | Total Model $R^2$ | Additional Variance Explained by Pathway |
|---|---|---|---|
| Grade K BOY | Grade 1 EOY | .68 | .32 |
| Grade 1 BOY | Grade 2 EOY | .48 | .11 |
| Grade 2 BOY | Grade 3 EOY | .52 | .14 |
| Grade 3 BOY | Grade 4 EOY | .61 | .16 |
| Grade 4 BOY | Grade 5 EOY | .66 | .10 |
| Grade 5 BOY | Grade 6 EOY | .61 | .14 |

*Note.* BOY = beginning of year. EOY = end of year.

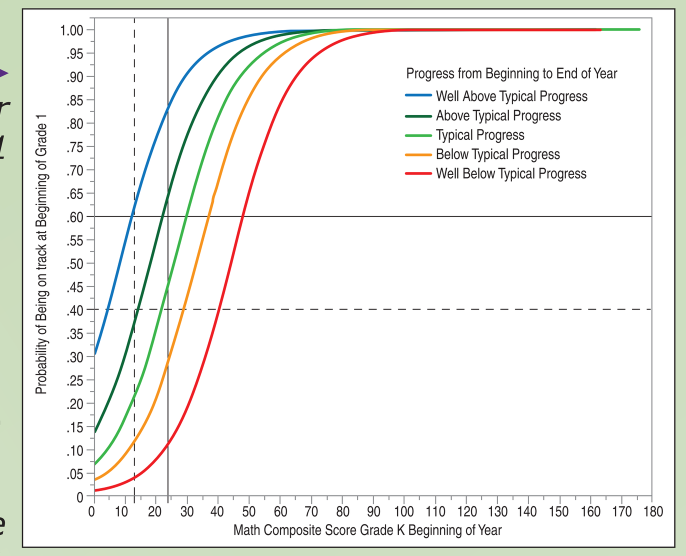
## THE POWER OF PATHWAYS (Continued)

### Results

Below are the results of the analysis. Pathways of Progress added substantial variance explained, above and beyond a student's initial starting point. In most grades, the additional variance explained is between 11 and 16%. However, astoundingly, in kindergarten the additional variance is over 30%. Clearly, these results indicate that Pathways of Progress provided powerful predictive validity for student math skills.

Additionally, we predicted the probability of students being At or Above Benchmark at the same outcome time point (e.g., the probability of students being At or Above Benchmark at end of first grade, given kindergarten pathways). Below are figures for each grade that we examined. There is clearly a trend for students who are on higher pathways to have a higher probability of meeting later Math goals, consistent with the above findings.

**Figure 3 ▶**
*Predicting Students At or Above Benchmark at End of Grade 1*



**◀ Figure 4**
*Predicting Students At or Above Benchmark at End of Grade 2*

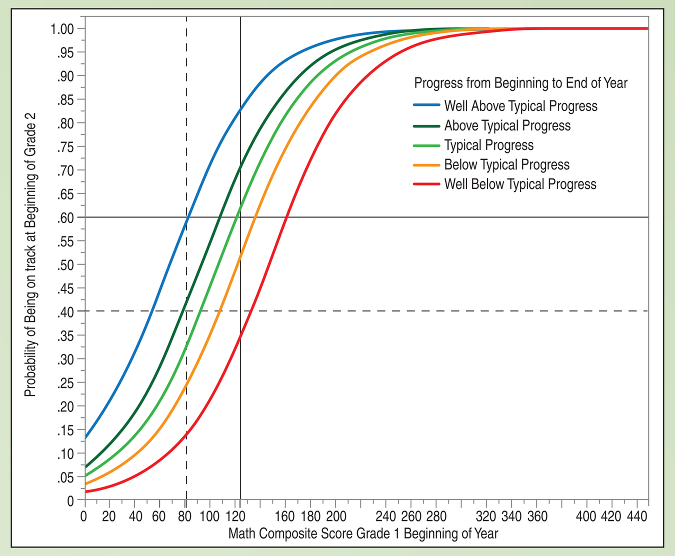
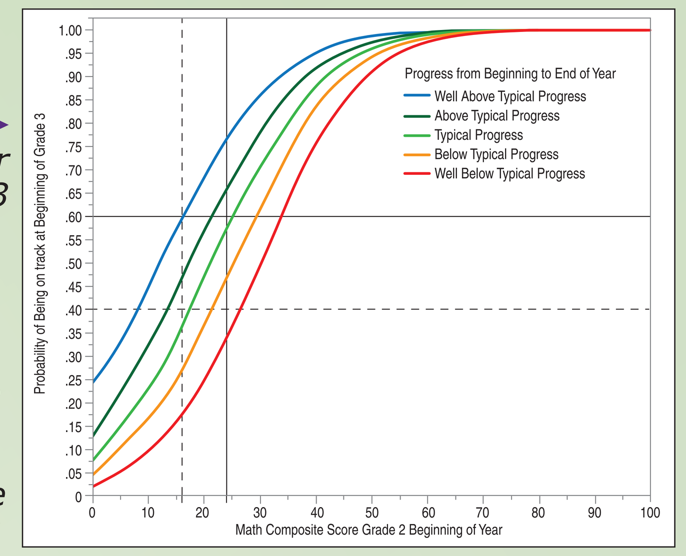


**Figure 5 ▶**
*Predicting Students At or Above Benchmark at End of Grade 3*



**◀ Figure 6**
*Predicting Students At or Above Benchmark at End of Grade 4*

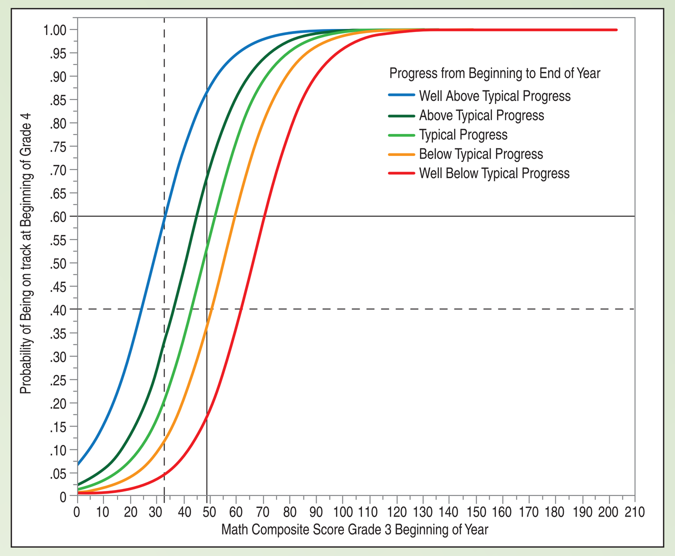
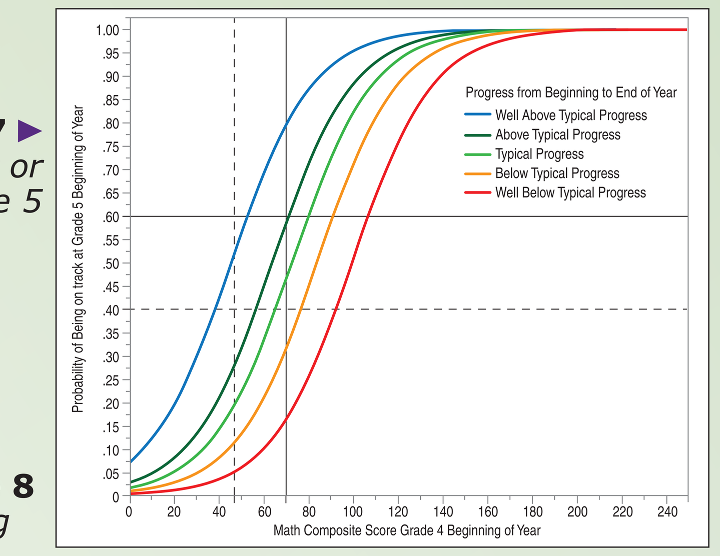
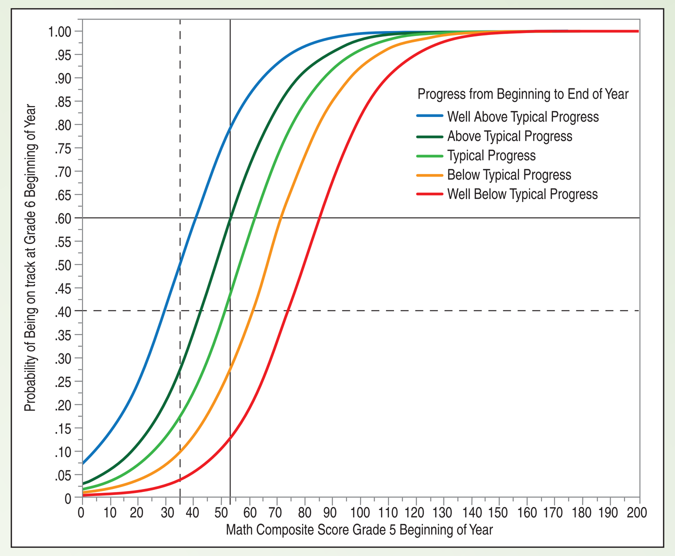


**Figure 7 ▶**
*Predicting Students At or Above Benchmark at End of Grade 5*



**◀ Figure 8**
*Predicting Students At or Above Benchmark at End of Grade 6*



## CONCLUSION

The present study sought to create and examine the predictive power of Pathways of Progress for Acadience Math. Pathways of Progress allow for the setting of goals that are both ambitious and attainable for students given their beginning-of-year Acadience Math scores. The process of creating Pathways of Progress were given and the incremental validity of pathways was assessed. The results indicate that the Pathways of Progress add predictive utility to later MCS by explaining additional variance by the end of the following year. Because pathways are created based on a student's beginning-of-year MCS, they are uncorrelated to which pathway a student is on, and therefore the predictive utility of pathways is independent of initial math skills. This demonstrates that pathways of Progress can be successfully utilized to identify students who are or are not on track to meet later math outcomes.



References available by contacting the authors.